Cost-effective Cooperative RTK positioning for rowing boats

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BIOGRAPHIES

Jane Jean Kiam graduated in 2013 as an engineer from Télécom Bretagne in France and received a M.Sc. degree from the Technische Universität München (TUM) in Germany. She did her Master Thesis on "Low-cost GPS-based Compass with Reliable Ambiguity Resolution and Cycle Slip Correction" at the Institute for Communications and Navigation of TUM. Her work was awarded the VDI Master Thesis Prize 2013. Since July 2013, she has joined Advanced Navigation Solutions - ANavS. Her job scope focuses on the development of Real Time Kinematic and attitude determination solutions with single-frequency low-cost GPS receivers. She is also an external doctoral student at the TUM and her research interest is precise point positioning with GNSS and INS.

Juan Manuel Cárdenas studied aerospace engineering at the FH Aachen University in Germany and developed his diploma thesis at the German Aerospace Center (DLR). He received two scholarships for his M.Sc. studies in navigation and satellite applications at the Technische Universität München in Germany and graduated with distinction. He worked as a scientific consultant for the Colombian government in two tenders for the acquisition of a large communications satellite and in the development of technical regulations for satellite navigation, Earth observation and space technology transfer. Juan gave two keynote lectures about satellite technology for the Bolivian Air Force in 2012 and was selected as one of the 100 successful Colombians abroad in 2013 in an event hosted by the Colombian president. He is one of the founders and project managers of Advanced Navigation Solutions - ANavS.

Patrick Henkel received his Bachelor, Master and PhD degrees from the Technische Universität München, Munich, Germany. In 2010, he graduated with a PhD thesis on reliable carrier phase positioning, and received with "summa cum laude" the highest distinction. Patrick is now working towards his habilitation in the field of precise point positioning. He visited the Mathematical Geodesy and Positioning group at TU Delft in 2007, and the GPS Lab at Stanford University in 2008 and 2010. Patrick received the Pierre Contensou Gold Medal in 2007, the 1st prize in Bavaria at the European Satellite Navigation Competition in 2010, and the Vodafone Award for his dissertation in 2011. He is one of the founders and currently also the managing director of Advanced Navigation Solutions - ANavS.

ABSTRACT

Low-cost single-frequency GNSS receivers with patch antennas can track carrier phase measurements with millimeter- to centimeter level and can therefore provide position information comparable to geodetic grade receivers. However, in order to use carrier phase measurements for positioning, one has to first resolve the integer ambiguities, which is challenging in the case of low-cost receivers because code multipath errors can be tens of meters. A new Kalman filter is proposed in this paper for determining jointly the cooperative RTK float solution of multiple rover receivers with respect to a fix base station. The measurement model as well as the state space model used in the Kalman filter are carefully designed for simultaneous tracking of multiple rowing boats. The measurement model exploits the correlation between measurements and the state parameters are chosen without redundancy (i.e. common states are only estimated once). The characteristic periodic movement of a racing rowing boat is also exploited in this paper to correct for carrier phase measurement outliers, i.e. cycle slip. Extensive tests were conducted with low-cost single-frequency GPS receivers. Static and kinematic test results show that with the new cooperative RTK Kalman filter, float ambiguities converge much faster and integer ambiguities can be correctly fixed in less than a minute. With the movement model, cycle slips are reliably corrected during the precise tracking of the boats after ambiguity resolution.

INTRODUCTION

RTK with low-cost single frequency GNSS receivers and patch antennas is quite attractive for rowing boat competitions. Today, cameras are used in rowing boat competitions to determine the time of arrival and thereby, the order of athletes. These cameras have one disadvantage: They are mounted at fixed points. As a rowing regatta course is typically 2 km long, the cameras are not capable of providing any timing information for most of the time. In this paper, we describe an RTK method for low-cost GNSS receivers and patch antennas for rowing boat competitions. As shown in Fig. 1, one antenna is typically installed at a fixed point, e.g. the finish tower while the other antennas are mounted on the rowing boats.

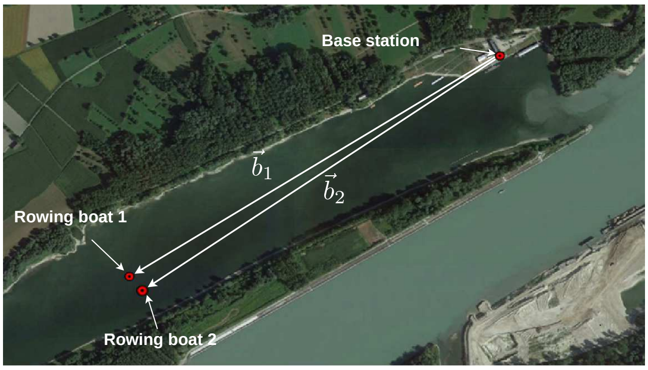


Fig. 1: Regatta course of Linz-Ottensheim, Austria where 2013 World Rowing U23 Championships took place. One antenna (base station) is mounted on the finish tower and one antenna (rover) is mounted on each rowing boat. Each base-rover pair forms a baseline vector.

In order to achieve centimeter-level accuracy with low-cost GNSS single-frequency receiver, one has to exploit the carrier phase measurements as code measurements are too noisy due to multipath delays. To track the rowing boats, we use double difference measurements to determine the baseline vectors between the finish tower and the boats. As the receiver clock offsets of low-cost GNSS receivers are in the order of milliseconds, the movement of the satellites within the time difference of the receiver clock offsets is no longer negligible. We use a synchronization correction [2], which corrects for this issue to restore the integer property of double difference measurements for low-cost GNSS receivers. The carrier phase integer ambiguities are then determined in three steps [1]: first, a float solution is determined by using code and carrier phase measurements and by disregarding the integer property of ambiguities. Subsequently, the float solution is mapped to an integer one by a tree search with integer decorrelation. Finally, the fixed baseline solution is determined.

A standard world championship rowing boat race takes in average 5 to 8 minutes; a quick ambiguity fixing is hence essential to ensure that rowing boats are tracked with centimeter accuracy in most part of the race. In this paper, we perform a cooperative RTK positioning using Kalman filter for multiple baselines, i.e. we exploit the correlation introduced by using the measurements of the finish tower for the double differences derived from different base-rover receiver pairs. Furthermore, since all rover receivers move on the same two-dimensional plane, we assume that all baselines share a common height component in the local East-North-Up coordinate frame and that the velocity in the Up-direction is negligible. By doing so, the Kalman Filter used for cooperative RTK has fewer state parameters to estimate while the number of available measurements remains unchanged. If the orientation of the racing course is perfectly known, the RTK positioning can be performed in a local coordinate frame defined by along-track (A), cross-track (C) and up (U) directions, which enables us to adapt more appropriately the process noise of the Kalman filter. The carefully adapted Kalman Filter allows a fast convergence of the float ambiguities and therefore ambiguity resolution with LAMBDA can be performed earlier in the race.

After the carrier phase integer ambiguities are fixed, rover receivers are tracked with centimeter-level accuracy. However, due to frequent cycle slips, jumps of a factor of half a cycle (i.e. 9 cm for GPS L1 band) can occur. In this paper, we develop a reliable cycle slip correction for cases where only GNSS measurements are available. We exploit the characteristic periodic dynamics of rowing boats to derive a priori baseline knowledge. With the latter, a Maximum A Posteriori probability estimator can be used, followed by an integer search to determine the number of cycle slips [4].

A more advanced movement model is also proposed in this paper which can be directly integrated into the cooperative RTK Kalman Filter used for determining float solution. This approach scales the velocity of the rowing boat according to a nominal velocity using three parameters in the rowing boat movement model namely phase of stroke, period of stroke and multiplier of nominal speed.

MEASUREMENT MODEL

Model of double difference (DD) code and phase measurements

The double difference (DD) carrier phase measurements for satellite pair $\{k, l\}$ and base-rover pair $\{B, r\}$ can be modeled at time t_n as

$$\begin{aligned}\lambda\varphi_{\mathrm{B}r}^{kl}(t_n) =& (\vec{e}^{\,kl}(t_n))^{\mathrm{T}}\vec{b}_{\mathrm{B}r}(t_n) + c_{\mathrm{B}r}^{kl}(t_n) \\ &+ \lambda N_{\mathrm{B}r}^{kl} + \lambda/2\Delta N_{\mathrm{B}r}^{kl}(t_n) + \varepsilon_{\varphi_{\mathrm{B}r}^{kl}}(t_n), \quad (1)\end{aligned}$$

with the wavelength λ, the unit vector $\vec{e}^{\,kl}$ pointing from the reference receiver to the rover receiver, the DD synchronization correction $c_{\mathrm{B}r}^{kl}$, the integer ambiguity $\lambda N_{\mathrm{B}r}^{kl}$, the cycle slips $\Delta N_{\mathrm{B}r}^{kl}$ and the carrier phase measurement noise $\varepsilon_{\varphi_{\mathrm{B}r}^{kl}}$.

The DD synchronization correction is required for low-cost GPS receivers as the satellite movement within the period of the differential receiver clock offset can be no longer neglected. In order to restore the integer property of the DD ambiguities, Henkel et al. developed a synchronization correction in [2], which solely depends on the receiver clock offset estimates, the satellite-receiver line of sight vectors, and the receiver and satellite movement within the period of the differential receiver clock offset. The DD synchronization correction is given by

$$\begin{aligned} & c^{kl}_{\mathrm{B}r}(t_n+\delta\tau_{\mathrm{B}}, t+\delta\tau_r) \\ & = (r^k_{\mathrm{B}}(t_n+\delta\tau_{\mathrm{B}}) - r^l_{\mathrm{B}}(t_n+\delta\tau_{\mathrm{B}})) \\ & \quad - (r^k_{\mathrm{B}}(t_n+\delta\tau_r) - r^l_{\mathrm{B}}(t_n+\delta\tau_r)), \end{aligned} \tag{2}$$

wherer^k_{B} is the range between satellite k and the base station, and can be expressed as follows:

$$\begin{aligned} & r^k_{\mathrm{B}}(t_n+\delta\tau_r) \\ & = (\vec{e}^{\,k}_{\mathrm{B}}(t_n+\delta\tau_r))^{\mathrm{T}} \cdot \\ & \quad \left(\vec{x}_{\mathrm{B}}(t_n+\delta\tau_r) - \vec{x}^{\,k}(t_n+\delta\tau_r-\Delta\tau^k_r)\right), \end{aligned} \tag{3}$$

with the unit vector $\vec{e}^{\,k}_r$ pointing from satellite k to receiver r, the base station absolute position $\vec{x}_{\mathrm{B}}$ and the delay $\Delta\tau^k_r$ between the transmission time and the received time.

A similar DD measurement model can also be applied to code measurements, i.e.

$$\begin{aligned} \rho^{kl}_{\mathrm{B}r}(t_n) =& (\vec{e}^{\,kl}(t_n))^{\mathrm{T}}\vec{b}_{\mathrm{B}r}(t_n) + c^{kl}_{\mathrm{B}r}(t_n) \\ & + m_{\rho^{kl}_{\mathrm{B}r}}(t_n) + \varepsilon_{\rho^{kl}_{\mathrm{B}r}}(t_n), \end{aligned} \tag{4}$$

with the code multipath delay $m_{\rho^{kl}_{\mathrm{B}r}}(t_n)$ and the code noise $\varepsilon_{\rho^{kl}_{\mathrm{B}r}}$.

Rowing boats in a regatta race have well defined movement and therefore the use of a Kalman filter is obviously beneficial to obtain an optimized estimate of the system state. In the case where several rowing boats have to be tracked simultaneously, a filter which processes jointly DD measurements of all baselines can exploit the correlation between DD measurements to improve precision of the baseline estimates. Furthermore, as rowing boats move on the same two-dimensional plane, all baseline vectors pointing from the base station to the boats share the same height component in the local East-North-Up (ENU) coordinate frame. A *cooperative RTK Kalman filter* is therefore constructed such that it considers

- correlation between DD measurements of different baselines due to common base station and common satellite (e.g. reference satellite),
- one common Up-component for all baselines.

After applying the DD synchronization correction, the matrix-vector representation of DD phase and code measurements at t_n can be written as

$$z_n = \begin{pmatrix} \lambda\varphi_n \\ \rho_n \end{pmatrix} = \begin{pmatrix} \tilde{H}_n & 0 & \lambda\cdot I & 0 \\ \tilde{H}_n & 0 & 0 & 1 \end{pmatrix} x_n + v_n, \tag{5}$$

where $\tilde{H}_n$ maps the baselines to the DD measurements, $\lambda\varphi_n$ and ρ_n stack respectively DD phase measurements and DD code measurements of all receiver pairs $\{B, r\}$ in a column vector:

$$\lambda\varphi_n = \begin{pmatrix} \lambda\varphi_{\mathrm{B}r} \\ \vdots \\ \lambda\varphi_{\mathrm{B}R} \end{pmatrix}_n \tag{6}$$

with

$$\lambda\varphi_{\mathrm{B}r,n} = \begin{pmatrix} \lambda\varphi^{12}_{\mathrm{B}r} - c^{12}_{\mathrm{B}r} \\ \vdots \\ \lambda\varphi^{1K}_{\mathrm{B}r} - c^{1K}_{\mathrm{B}r} \end{pmatrix}_n . \tag{7}$$

Note that the time-dependency of measurements is omitted in the matrix-vector representation for better readability.

The state vector in Eq. 5 is parameterized in local coordinate frame. During a rowing boat race, the boats intend to move in a line parallel to the rowing course as any movement in cross-track direction would result in a loss of time. It is hence optimal to describe the position and velocity of the rowing boat in a local coordinate frame defined by the along-track (A), the cross-track (C) and the up (U) direction, i.e.

$$x_n = \left(\vec{b}^{\mathrm{T}}_{\mathrm{AC}}, b^{\mathrm{T}}_{\mathrm{U}}, \dot{\vec{b}}^{\mathrm{T}}_{\mathrm{AC}}, N^{\mathrm{T}}, m^{\mathrm{T}}_{\rho}\right)^{\mathrm{T}}_n, \tag{8}$$

where $\vec{b}_{\mathrm{AC}}$ stacks the baselines in along-track and cross-track direction and $\dot{\vec{b}}_{\mathrm{AC}}$ stacks the baseline rates, i.e.

$$\vec{b}_{\mathrm{AC},n} = \begin{pmatrix} \vec{b}_{\mathrm{AC,B1}} \\ \vdots \\ \vec{b}_{\mathrm{AC,B}R} \end{pmatrix}_n \quad \text{and} \quad \dot{\vec{b}}_{\mathrm{AC},n} = \begin{pmatrix} \dot{\vec{b}}_{\mathrm{AC,B1}} \\ \vdots \\ \dot{\vec{b}}_{\mathrm{AC,B}R} \end{pmatrix}_n, \tag{9}$$

while N and m_ρ stack the DD phase ambiguities and the DD code multipath delays respectively, i.e.

$$N = \begin{pmatrix} N_{\mathrm{B1}} \\ \vdots \\ N_{\mathrm{B}R} \end{pmatrix} \quad \text{and} \quad m_\rho = \begin{pmatrix} m_{\rho_{\mathrm{B1}}} \\ \vdots \\ m_{\rho_{\mathrm{B}R}} \end{pmatrix}. \tag{10}$$

The baseline in ACU coordinates is related to the baseline in Earth-Center-Earth-Fixed (ECEF) coordinates by the following expression:

$$\begin{pmatrix} \vec{b}_{\mathrm{AC}} \\ b_U \end{pmatrix} = R_z(\psi+\pi/2)\cdot R_{\mathrm{L}}\cdot \vec{b}_{\mathrm{ECEF}}, \tag{11}$$

where

$$R_{\mathrm{L}} = R_x(\pi/2-\varphi_{\mathrm{lat}})\cdot R_z(\pi/2+\lambda_{\mathrm{lon}}), \tag{12}$$

with ψ being the heading of the regatta course calculated clock-wise from the North-direction, φ_{lat} being the latitude and λ_{lon} being the longitude. Note however that if the orientation of the rowing course is unknown, the position and velocity can also be modeled in the East-North-Up local coordinate frame. As mentioned earlier, the rowing boats to be tracked move on the same 2-D plane. It is therefore justified to estimate only one baseline Up-component b_{U} for all receiver pairs. Besides, the velocity in the Up-direction is disregarded in the Kalman filter.

v_n in Eq. 5 is the DD phase and code measurement noise, which is assumed to be zero-mean Gaussian white noise with covariance matrix R_n. The noise statistics can be estimated with an elevation based decreasing exponential model as proposed in [7]. While constructing the covariance matrix with the model, the correlation between DD measurements must also be considered:

$$\begin{aligned} v_n &\sim N(0, R_n), \quad \text{where} \\ (R_n)_{i,j} &= \mathrm{cov}(\varphi_{\mathrm{B}i}(t_n), \varphi_{\mathrm{B}j}(t_n)), \\ (R_n)_{R+i',R+j'} &= \mathrm{cov}(\rho_{\mathrm{B}i'}(t_n), \rho_{\mathrm{B}j'}(t_n)), \\ \text{for} \quad & 1 \leq \{i, j, i', j'\} \leq R. \end{aligned} \tag{13}$$

The measurement noise covariance matrix can be partitioned into a diagonal block matrix with each block being a dense matrix.

STATE SPACE MODEL

Constant linear acceleration assumption

A Gauss-Markov process can be used to model the movement of a rowing boat, i.e.

$$\underbrace{\begin{pmatrix} \vec{b}_{\mathrm{AC},n} \\ b_{\mathrm{U},n} \\ \dot{\vec{b}}_{\mathrm{AC},n} \\ \ddot{\vec{b}}_{\mathrm{AC},n} \\ N_n \\ m_{\rho,n} \end{pmatrix}}_{x_n} = \underbrace{\left(\begin{array}{cc|cc|cc} 1 & & \Delta t \cdot 1 & \frac{\Delta t^2}{2} \cdot 1 & & \\ & 1 & & & & \\ \hline & & 1 & \Delta t \cdot 1 & & \\ & & & 1 & & \\ \hline & & & & 1 & \\ & & & & & 1 \end{array}\right)}_{\mathbf{\Phi}} \underbrace{\begin{pmatrix} \vec{b}_{\mathrm{AC},n-1} \\ b_{\mathrm{U},n-1} \\ \dot{\vec{b}}_{\mathrm{AC},n-1} \\ \ddot{\vec{b}}_{\mathrm{AC},n-1} \\ N_{n-1} \\ m_{\rho,n-1} \end{pmatrix}}_{x_{n-1}} + \underbrace{\begin{pmatrix} w_{\vec{b}_{\mathrm{AC},n}} \\ w_{b_{\mathrm{U},n}} \\ w_{\dot{\vec{b}}_{\mathrm{AC},n}} \\ w_{\ddot{\vec{b}}_{\mathrm{AC},n}} \\ w_{N_n} \\ w_{m_{\rho,n}} \end{pmatrix}}_{\boldsymbol{w}_n}, \tag{14}$$

where Δt denotes the time interval between the current and the previous states. In this state-space model, we model the change of acceleration as white Gaussian noise. $b_{\mathrm{U},n}$ and N_n are assumed to remain constant over time, while $m_{\rho,n}$ follows a random walk process.

Periodic dynamics of a racing rowing boat

Rowing boat is a cyclic sport [6] in which the periodicity of the strokes results in the periodic pattern of the rowing boat dynamics. Figure 2 depicts respectively the periodic pattern of a racing rowing boat along-track velocity and along-track acceleration. Note however, that the dynamics are not derived with the center of mass of the boat but with the center of the patch antenna mounted on the boat.

Rowing boat accelerates when a reaction force is exerted on the oar and decelerates when the oar is extracted. Fig. 2 illustrates the dynamics of a rowing boat within one stroke. A stroke cycle can in general be subdivided into two main phases, namely *recovery* and *drive* phases. The latter begins when the oar blade is immersed in water and ends when the blade is removed from water; recovery phase starts once the blade is removed from the water and subsequently, the speed of the boat decreases. Detailed analysis of the anatomy of a stoke cycle can be found in [6].

Precise Modeling of Characteristic Velocity Pattern

In this subsection, we derive a model for the characteristic velocity pattern using the velocity estimates from multiple strokes. We subdivide the characteristic velocity pattern of a stroke into p sections. The velocity pattern of each section is described by a polynomial of second order. The *continuity* of the velocity pattern and of its derivative at the *boundaries* of the p sections is ensured by introducing constraints on the polynomial coefficients.

We model the velocity measurements of p phases using measurements from n_p epochs for the p-th phase as in Eq. 15, with

$$v^{(i)} = \begin{pmatrix} v^i(t_1^i) \\ \vdots \\ v^i(t_{n_i}^i) \end{pmatrix}, \quad \alpha^{(i)} = \begin{pmatrix} \alpha_0^i \\ \alpha_1^i \\ \alpha_2^i \end{pmatrix}, \tag{16}$$

$$U^{(i)} = \left(\begin{array}{c|c|c} 1 & t_1^i - t_0 & (t_1^i - t_0)^2 \\ \vdots & \vdots & \vdots \\ 1 & t_{n_i}^i - t_0 & (t_{n_i}^i - t_0)^2 \end{array}\right), \tag{17}$$

and the coefficients for the continuity requirements

$$\begin{aligned} f^{(i)} &= \left(1, \quad \sum_{j=1}^{i} T_j, \quad \left(\sum_{j=1}^{i} T_j\right)^2\right), \\ g^{(i)} &= \left(0, \quad 1, \quad \left(\sum_{j=1}^{i} T_j\right)\right), \end{aligned} \tag{18}$$

$$
\left(\begin{array}{c} v^{(1)} \\ v^{(2)} \\ \vdots \\ v^{(p-1)} \\ v^{(p)} \\ \hline 0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ 0 \end{array}\right) = \left(\begin{array}{c|c|c|c|c} U^{(1)} & & & & \\ & U^{(2)} & & & \\ & & \ddots & & \\ & & & U^{(p-1)} & \\ & & & & U^{(p)} \\ \hline (f^{(1)})^{\mathrm{T}} & -(f^{(1)})^{\mathrm{T}} & & & \\ & & \ddots & & \\ & & & (f^{(p)})^{\mathrm{T}} & -(f^{(p)})^{\mathrm{T}} \\ \hline (g^{(1)})^{\mathrm{T}} & -(g^{(1)})^{\mathrm{T}} & & & \\ & & \ddots & & \\ & & & (g^{(p)})^{\mathrm{T}} & -(g^{(p)})^{\mathrm{T}} \end{array}\right) \left(\begin{array}{c} \alpha^{(1)} \\ \alpha^{(2)} \\ \vdots \\ \alpha^{(p-1)} \\ \alpha^{(p)} \end{array}\right) + \left(\begin{array}{c} \eta_{v^{(1)}} \\ \eta_{v^{(2)}} \\ \vdots \\ \eta_{v^{(p-1)}} \\ \eta_{v^{(p)}} \\ \hline 0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ 0 \end{array}\right), \quad (15)
$$

and T_j being the period of the j-th phase. The polynomial coefficients are determined by least-squares estimation as

$$
\left(\begin{array}{c} \alpha^{(1)} \\ \alpha^{(2)} \\ \vdots \\ \alpha^{(p-1)} \\ \alpha^{(p)} \end{array}\right) = \left(B_\alpha^{\mathrm{T}} \Sigma_\alpha^{-1} B_\alpha\right)^{-1} B_\alpha^{\mathrm{T}} \Sigma_\alpha^{-1} \left(\begin{array}{c} v^{(1)} \\ v^{(2)} \\ \vdots \\ v^{(p-1)} \\ v^{(p)} \\ \hline 0^{p\times 1} \\ 0^{p\times 1} \end{array}\right), \quad (19)
$$

where B_α is implicity defined by Eq. (15). The measurement covariance matrix Σ_α enables a weighting of velocity measurements. However, the main purpose of the weighting matrix is to ensure the fulfillment of the continuity requirements. Therefore, we set the variances of the "zero"-measurements to 10^{-6} and of all other measurements to 1. Fig. 3 shows a velocity model derived by dissecting a stroke into sections with each section carefully modeled with a second order polynomial. The continuity of the polynomial of one section to the next is ensured by the coefficients in Eq. 18.

We tested the coherence of the velocity model with real measurement by predicting the positions of a rowing at the subsequent epochs given a known starting point. Fig. 4 compares the relative positions estimated using the velocity model and the relative positions estimated using DD carrier phase measurements. The relative positions estimated with the velocity model (in red) are comparable to the relative positions estimated with DD phase measurements (in blue). One can hence benefit from the model to estimate position information independent from the measurements, which can be useful in case of missing measurements due to data losses in the communication link and in case of erroneous measurements due to cycle slips.

Optimized state space model

The state space model described in Eq. 14 does not fully exploit the characteristic movement of rowing boats. In this subsection, an optimized state space model which exploits the cyclic movements of a racing rowing boat is proposed. The velocity of the rowing boat follows a periodic pattern, it is hence appropriate to derive the velocity using stroke parameters, namely phase of stroke α, period of stroke T and multiplier of nominal speed γ:

$$
\dot{\vec{b}}_{\mathrm{AC,B}r}(t) = f(\alpha, T, \gamma) = \gamma(t) \cdot \vec{v_{\mathrm{M}}}(\alpha(t) \cdot T(t)), \quad (20)
$$

where $\vec{v}_{\mathrm{M}}$ is a continuous model function derived from the periodic velocity at the past epochs of the rowing boat. The underlying model assumes that the velocity pattern is well-defined but allows at the same time a variable stroke period as well as variable speed. The flexibility in stroke period and speed is important as these parameters differ from one rower/ rowing team to another and depending on the strategy, at different phases of a race, the speed can vary. The velocity of the boat is therefore no longer directly estimated but derived from the movement model with estimated stroke parameters from the extended Kalman filter. We use the following state vector:

$$
x_n = \left(\vec{b}_{\mathrm{AC}}^{\mathrm{T}}, b_{\mathrm{U}}^{\mathrm{T}}, \alpha', T, \gamma, N^{\mathrm{T}}, m_\rho^{\mathrm{T}}\right)_n^{\mathrm{T}}, \quad (21)
$$

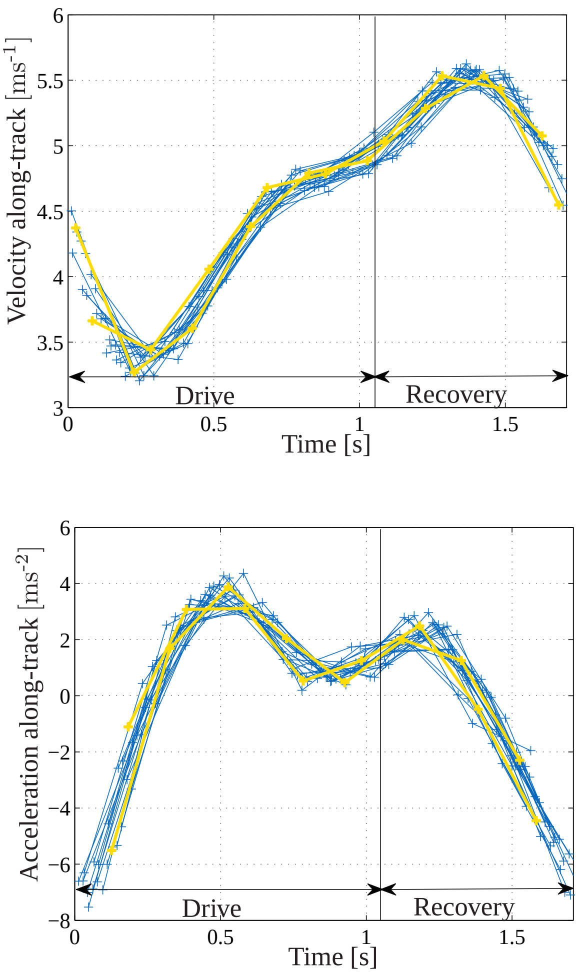
where $\alpha(t_n) = \mathrm{mod}\,(\alpha'(t_n), 1)$. The Gauss-Markov processes of the baseline and the stroke parameters are described as follows:

$$
\begin{aligned}
\alpha'(t_n) &= \alpha'(t_{n-1}) + \Delta t / T(t_{n-1}) + w_{\alpha'}(t_n), && (22) \\
T(t_n) &= T(t_{n-1}) + w_T(t_n), && (23) \\
\gamma(t_n) &= \gamma(t_{n-1}) + w_\gamma(t_n). && (24)
\end{aligned}
$$

CYCLE SLIP CORRECTION

Maximum a posteriori probability estimation with a priori baseline information

Ambiguity resolution can be performed with Teunissen's LAMBDA method [1] once the Kalman filter estimated



(a)

Fig. 2: a) Velocity of rowing boat in along-track direction; b) Acceleration of rowing boat in along-track direction. The velocity and acceleration are shown for 20 strokes with the first and last being additionally marked. One can observe a characteristic pattern and high repeatability. The period of a stroke varies by only 0.1 s over 20 strokes. The average period is 1.71 s.

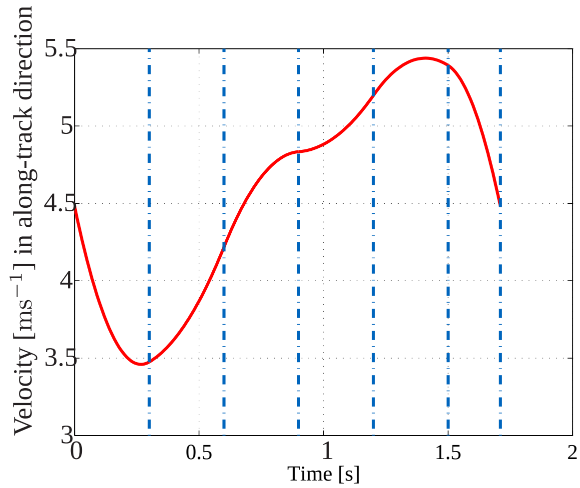


Fig. 3: Velocity model of one stroke which is subdivided into 6 sections. Each section is between two vertical blue lines and is described by a second order polynomial. The transition of the polynomial between sections is continuous.

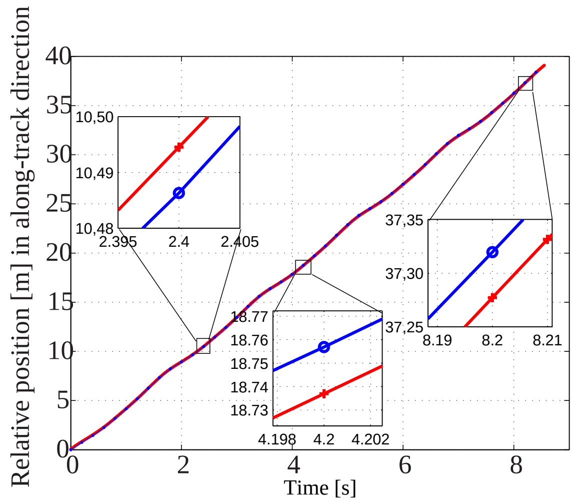


Fig. 4: The red curve shows the positions of the boat with respect to the initial position estimated using the velocity model. The blue curve shows the positions of the boat with respect to the initial position estimated using DD carrier phase measurements and RTK. The difference between both estimations is less than 10 cm.

float ambiguities have converged. Fixed solution is thereafter used to track the rowing boats. Low-cost receivers however experience frequent half cycle slips which, if left uncorrected, result in an accumulative estimation error. A reliable cycle slip correction method which uses a maximum a posteriori (MAP) probability estimator and an a priori baseline knowledge is proposed in [4] and [5].

The joint estimation of the baseline and cycle slips with a MAP probability estimator and known fixed DD phase measurements can be written as follows:

$$\begin{pmatrix} \check{\vec{b}}_{\mathrm{AC,Br}} \\ \Delta\check{N} \end{pmatrix} = \arg \max_{\substack{\vec{b}_{\mathrm{AC,Br}} \in \mathbb{R}^{2\times 1} \\ \Delta N \in \mathbb{Z}^{q\times 1}}} P(\vec{b}_{\mathrm{AC,Br}}, \Delta N | \varphi_{\mathrm{dd}} - A\check{N}). \quad (25)$$

Similar to LAMBDA method, cycle slip can be first determined in its float form and subsequently fixed to an integer with a discrete search. With the rule of Bayes, the probability in the above equation can be rewritten as

$$P(\vec{b}_{\text{AC,Br}}, \Delta N | \varphi_{\text{dd}} - A\check{N}) = \frac{P(\varphi_{\text{dd}} - A\check{N} | \vec{b}_{\text{AC,Br}}, \Delta N) P(\vec{b}_{\text{AC,Br}}, \Delta N)}{P(\varphi_{\text{dd}} - A\check{N})}. \quad (26)$$

$P(\varphi_{\text{dd}} - A\check{N})$ is marginal and therefore does not play a role in the maximization. By assuming Gaussian distribution for the measurement noise as well as for the distribution of the a priori baseline information, and by assuming that measurements and baseline a priori information are not correlated, the maximization can be rewritten as a minimization as follows:

$$\begin{pmatrix} \check{\vec{b}}_{\text{AC,B}r} \\ \Delta\check{N} \end{pmatrix} = \arg \min_{\substack{\vec{b}_{\text{AC,B}r} \in \mathbb{R}^{2\times 1} \\ \Delta N \in \mathbb{Z}^{q\times 1}}} \left\| z - \tilde{H}\vec{b}_{\text{AC,B}r} - A_{\text{CS}}\Delta N \right\|^2_{\Sigma_z^{-1}}, \quad (27)$$

with z being the extended measurement vector which includes the a priori baseline derived from the movement model and $\tilde{H}$ being the matrix which maps the estimates to the extended measurement vector, i.e.

$$z = \begin{pmatrix} \varphi_{\text{dd}} - A\check{N} \\ \vec{b}_{\text{AC,ap}} \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} H \\ I_2 \end{pmatrix}, \quad (28)$$

and A_{CS} being a matrix which maps cycle slips to z.

Henkel and Oku used in [3] baseline a priori knowledge derived from inertial sensors to correct for cycle slips. In the absence of an external sensor providing independent measurements, $\vec{b}_{\text{AC,ap}}$ is normally difficult to determine in a way that it is not correlated with the DD measurements of the current epoch. However, in the case of rowing boats, the movement model can be used to predict an independent a priori baseline information with a simple equation of movement:

$$\vec{b}_{\text{AC,ap}}(t_n) = \hat{\vec{b}}_{\text{AC,Br}}(t_{n-1}) + \int_{t_{n-1}}^{t_n} \vec{v}_{\text{M}}(t + t_{\text{curr}}) \text{d}t + \int\int_{t_{n-1}}^{t_n} \vec{a}_{\text{M}}(t + t_{\text{curr}}) \text{d}t\text{d}t, \quad (29)$$

where $\vec{v}_{\text{M}}$ and $\vec{a}_{\text{M}}$ are the periodic model functions for rowing boat velocity and acceleration. t_{curr} in Eq. 29 is the translation which produces the largest convolution between the model function and the past samples, i.e.

$$\begin{aligned} t_{\text{curr}} &= \arg\max_{t'} \vec{v}_T(t) * \vec{v}_{\text{L,M}}(t - t') \\ &= \arg\max_{t'} \sum_{i=1}^{8} \vec{v}_{\text{AC,B}r}(t_{n-i}) \cdot \delta(t - t_{n-i}) * \vec{v}_{\text{M}}(t - t') \\ &= \arg\max_{t'} \sum_{i=1}^{8} \vec{v}_{\text{AC,B}r}(t_{n-i}) \cdot \vec{v}_{\text{M}}(t - t' - t_{n-i}), \end{aligned} \quad (30)$$

with $\vec{v}_{\text{T}}$ a function constructed with the previously estimated velocities:

$$\vec{v}_{\text{T}} = \sum_{i=1}^{8} \vec{v}_{\text{AC,B}r}(t_{n-i}) \cdot \delta(t - t_{n-i}), \quad (31)$$

where $\delta(t)$ here is a Dirac function.

Given the integer property of ΔN, Eq. 27 is an integer least-squares problem, which can be solved by first determining the floating cycle slip $\Delta\hat{N}$, then search within a volume χ^2. The search is performed sequentially for each DD phase cycle slip around the considered float ambiguity estimates, i.e.

$$\Delta\check{N}^k \leq \Delta\hat{N}^{k|1,\dots,k-1} + \sigma_{\Delta\hat{N}^{k|1,\dots,k-1}}\sqrt{\kappa}, \quad (32)$$

$$\Delta\check{N}^k \geq \Delta\hat{N}^{k|1,\dots,k-1} - \sigma_{\Delta\hat{N}^{k|1,\dots,k-1}}\sqrt{\kappa}, \quad (33)$$

where $\Delta\hat{N}^{k|1,\dots,k-1}$ is the conditional cycle slip and $\sigma_{\Delta\hat{N}^{l|1,\dots,l-1}}$ is the conditional standard deviation and κ is the multiplier of the conditional standard deviation as derived by Teunissen in [8]:

$$\kappa = \chi^2 - \|\check{b}_{\text{AC,B}r}(\Delta N) - b_{\text{AC,B}r}\|^2_{\Sigma^{-1}_{\check{b}_{\text{EN}}}} - \|P^{\perp}_{\bar{A}} P^{\perp}_{\tilde{H}} z\|^2_{\Sigma_z^{-1}} - \sum_{l=1}^{k-1} \left((\Delta N^l - \Delta\hat{N}^{l|1,\dots,l-1})^2 \right) / \left(\sigma^2_{\Delta\hat{N}^{l|1,\dots,l-1}} \right), \quad (34)$$

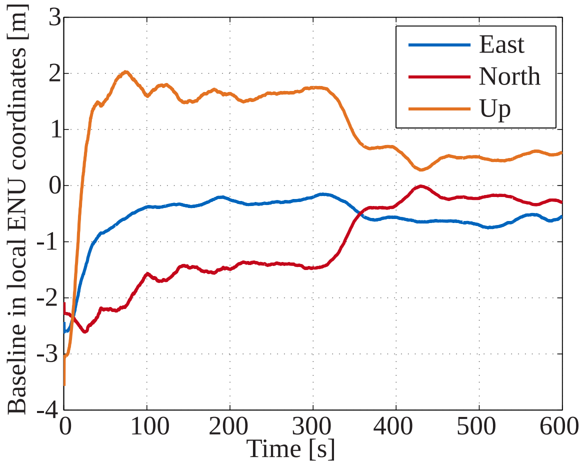
with $P^{\perp}_{\tilde{H}}$ being the orthogonal projector on the space of $\tilde{H}$ and $\bar{A} = P^{\perp}_{\tilde{H}} A_{\text{CS}}$.

TEST AND VALIDATION

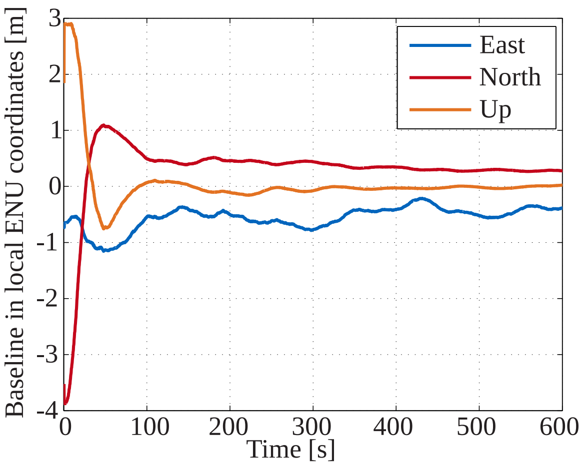
In this section, two tests are analyzed to compare the performance of a conventional Kalman filter for single-baseline estimation and the Kalman filter for cooperative RTK which assumes identical height between rovers and negligible movement in the Up-direction. The first test consist of 3 static patch antennas placed on the same level while the second test is a kinematic test performed within a rowing-boat competition (i.e. 2013 World Rowing U23 Championships) with one base-station patch antenna mounted on the top of the tower at the finishing line and two rover patch antennas, each mounted on a rowing boat. All tests are conducted using low-cost single-frequency GPS receivers u-blox LEA 6T with a 5 Hz sampling frequency.

Static rovers

The static test was conducted with 3 patch antennas placed on the same level on a rooftop with one antenna serving as the base station and two other antennas serving as rover antennas. We tested Kalman filter cooperative RTK with common-height assumption against a conventional Kalman filter. The main objective of the test is to compare the convergence speed of the float ambiguities for both filters. Fig. 5 shows the variation of one of the two baselines in



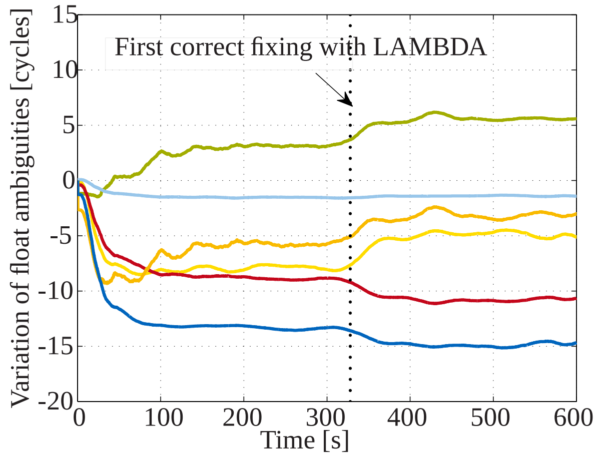
(a)



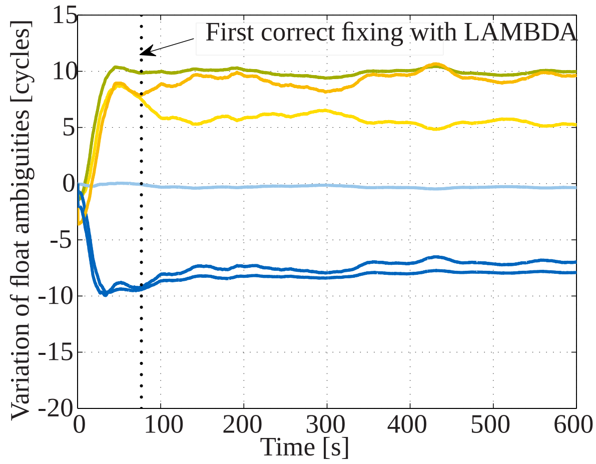
(b)

Fig. 5: a) Variation of a baseline estimated with Kalman filter for single-baseline estimation in local ENU coordinate frame; b) Variation of a baseline estimated with Kalman filter for cooperative RTK and identical height assumption in local ENU coordinate frame.

local ENU coordinates estimated with a conventional Kalman filter and a Kalman filter for cooperative RTK with common-height assumption respectively. One can observe that the convergence of the baseline happens after 5 minutes with a conventional Kalman filter. Furthermore, it converges to a wrong baseline, as the antennas were all placed on the same height level (i.e. $b_{\mathrm{U}} = 0$). On the contrary, the baseline converges much earlier with cooperative RTK Kalman filter. The performance of the cooperati-



(a)



(b)

Fig. 6: a) Variation of float ambiguities estimated with Kalman filter for single-baseline estimation in local ENU coordinate frame; b) Variation of float ambiguities estimated with Kalman filter for cooperative RTK and identical height assumption in local ENU coordinate frame.

ve RTK Kalman filter can also be proven by comparing the convergence of the float ambiguities estimated with the two Kalman filters (see Fig. 6). In this test, we try to fix the DD phase ambiguities with LAMBDA method [1] at each epoch and accept the first fixed integer ambiguities by considering the residual ratio between the best integer candidate and the second best integer candidate [1]. The first correct ambiguity resolution with cooperative RTK Kalman filter occurs within the first 100 seconds while with a conventional Kalman filter, one has to wait for more than 5 minutes before obtaining the first correct ambiguity fixing.

The benefit of a fast ambiguity fixing is essential for precise rowing boat tracking as a typical rowing boat race takes not more than 8 minutes.

Racing rowing boats

The cooperative RTK Kalman filter was tested with racing rowing boats at the 2013 World Rowing U23 Championship. The float ambiguities converge within the first 30 s and the boats are subsequently tracked with fixed DD phase measurements. Fig. 7 shows the course of the rowing boats which are determined with fixed DD phase measurements. With the cooperative RTK Kalman filter, ambiguities can be fixed faster and therefore, precise tracking can be performed for most part of the race.

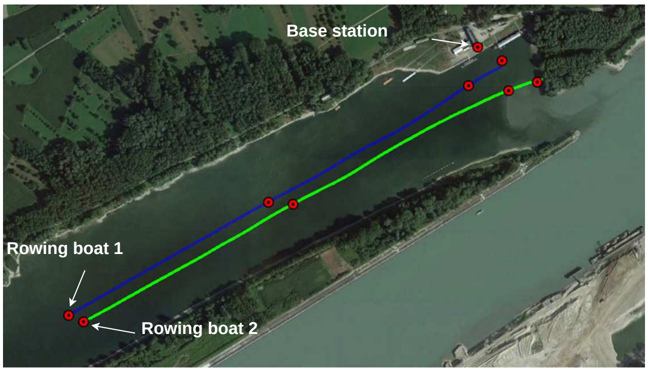


Fig. 7: Precise cooperative tracking of racing rowing boats after integer ambiguities are correctly resolved.

Fig. 8 depicts the fixed DD phase residuals during precise tracking of the racing rowing boats without and with cycle slip correction. The sudden jump in residuals is where cycle slip happens. With the a priori baseline derived from the periodic dynamics of a rowing boat, cycle slip as well as the baseline are jointly estimated with a MAP probability estimator and an integer search.

CONCLUSION

Kalman filter offers an efficient method to provide an optimal estimation based on measurements as well as the system dynamics model. If the models are carefully designed to adapt to the use-case, the convergence of the filter can be very efficient and accurate. In this paper, the carefully designed Kalman filter for rowing boat tracking is proven to be more efficient: the float solution converges much faster and therefore, integer ambiguities can be resolved earlier to provide precise tracking of the rowing boats. Besides, a cycle slip correction method based on a dynamics model derived from the characteristic periodic movement of a racing rowing boat is shown beneficial when GNSS-only measurements are available.

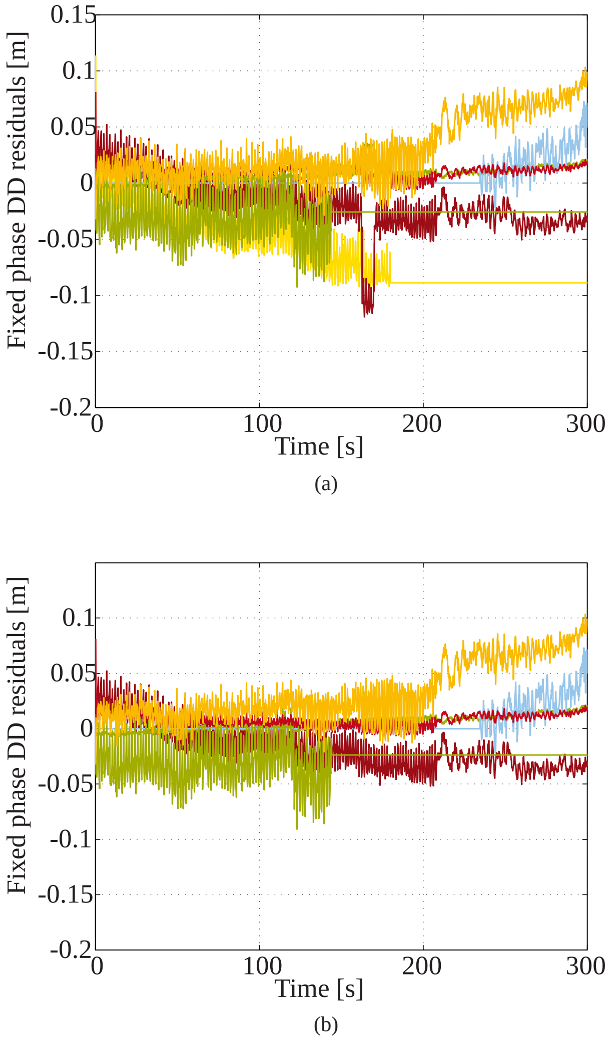


Fig. 8: a) Residuals of fixed DD phase measurements during precise tracking without cycle slip correction; b) Residuals of fixed DD phase measurements during precise tracking with cycle slip correction using MAP probability estimator and an a priori baseline knowledge derived from a movement model.

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REFERENCES

[1] P. Teunissen, The least-squares ambiguity decorrelation adjustment: a method for fast GPS ambiguity estimation, *J. of Geodesy*, vol. 70, pp. 65-82, 1995.

[2] P. Henkel and J.M. Cardenas, Method for determining

the baseline between two receivers, *European Patent Application, Appl. Nr.: EP 12 199 772.0*, Appl. date: 28.12.2012.

[3] P. Henkel and N. Oku, Cycle slip detection and correction for heading determination with low-cost GPS/INS receivers, *Internat. Association of Geodesy Symposia, Proc. of VIII Hotine-Marussi Symposium*, Rome, Italy, Jun. 2013.

[4] P. Henkel and J. Kiam, Maximum A Posteriori Probability Estimation of Integer Ambiguities and Baseline, *IEEE Proc. of 55-th ELMAR Symposium*, pp. 353-356, Zadar, Croatia, Sep. 2013.

[5] J.J. Kiam, Low-cost GPS-based Compass with Reliable Ambiguity Resolution and Cycle Slip Correction, *Masters thesis, Technische Universität München*, July 2013.

[6] V. Kleshnev, Temporal Analysis of Stroke Cycle in Rowing, *XXV International Symposium on Biomechanics in Sports*, 2007.

[7] P. Henkel and C. Günther, Partial integer decorrelation for optimum trade-off between variance reduction and bias amplification. *J. of Geodesy*, vol. 84, iss. 1, pp. 51-63, 2010.

[8] P. Teunissen, The least-squares ambiguity decorrelation adjustment: a method for fast GPS ambiguity estimation, *J. of Geodesy*, vol. 70, pp. 65-82, 1995.